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RESPONSE SURFACE METHODOLOGY AS A
SENSITIVITY TOOL IN DECISION ANALYSIS

THESIS

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THESIS

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DISCLAIMER STATEMENT

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U. S. Government.

PREFACE

I believe that today's decision-maker has a great need to understand the uncertainty involved in the analysis of decisions. This need has become increasingly complex and time-critical. With the appearance of a rapidly changing world, the military decision-maker is confronted with a potentially massive reduction in resources available to meet a variety of conditions. I believe this thesis has taken a step to provide the decision-maker a tool that reduces the resources required to adapt to these changes.

In doing this research, I have had help from many people. My thesis advisor, Lt Col Ken Bauer, provided many insights into the potential for RSM's use. His enthusiasm and optimism were much needed and greatly appreciated. My thesis reader, Col Parnell, was much more than a reader. He provided the basis of the decision analysis viewpoint from vast experience in that area. The advice from Lt Col Bauer and Col Parnell helped to make this work both demanding and enjoyable. I wish to thank them both. Finally, I could never have accomplished such a effort without the understanding and support of my family. To my wife, Deb, and my daughters, Chelsea and Kathryn, I thank you.

David A. Meyers

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ABSTRACT

The purpose of this study is to evaluate response surface methodology as a sensitivity analysis tool in the area of decision analysis. The advent of low-cost personal computer software, such as DPL^{TM}, has created an accessible tool with the ability to frame and solve influence diagrams for decision problems. The limited sensitivity analysis capabilities used in this field has created a need for improvement.

This study provides a comparison of current sensitivity analysis techniques vs those made possible through response surface methodology (RSM). Sensitivity analysis alternatives are demonstrated on a decision problem concerning the evaluation of force structure options for the Department of Defense. Sensitivity analysis is performed on both one-way and two-way perturbations of input variables.

The most significant contribution of this thesis is the effectiveness of RSM techniques for the sensitivity analysis of a decision problem. RSM is proven to be an effective and highly efficient approach. The combination of two analysis tools, the influence diagram solver and the RSM sensitivity analysis, has effectively saved the decision-maker valuable resources and increased the information made available. The resources saved are analysis and computer time. While the increased information includes multiple factor sensitivity analysis, as well as additional insights gained from the coefficients of the RSM equations. This capability will enhance the analysis of uncertainty in decision problems.

RESPONSE SURFACE METHODOLOGY AS A SENSITIVITY TOOL IN DECISION ANALYSIS

I. Introduction

Background

Decision makers in the Air Force are interested in an increased ability to evaluate uncertainty in decision analysis. Evaluating the possible alternatives is important, but so is analyzing the impact of uncertainty in the input variables. The decision maker wants to know what will happen if current inputs are off the mark. What if the threat to national survival has a 90% chance of being high in the next 20 years instead of the planned 70%? What if the cost of a buildup in force structure is higher than expected? Will this impact the optimal decision policy? In order to answer these questions with some level of confidence, the analyst must have the appropriate tools available. These tools should permit the analyst to recommend a course of action and provide information on the sensitivity of his recommendation to changes in the input parameters. This sensitivity information could involve changing a number of input parameters at the same time, which exceeds the current capability of decision analysis tools. The topic of this paper is a combination of methods, joining decision analysis and response surface methodology, which shows the potential to meet this need.

There are many types of approaches and models to analyze Air Force problems. The goal is to give the decision-maker dependable insight on the options. First, the analyst and decision-maker must agree on the problem. It is of utmost importance that the presentation is clear and to the point. This is where a good understandable graph or picture of the information is critical.

One modeling technique that captures and presents the key information is the influence diagram. Shachter, who created an algorithm to solve the influence diagram, defines them as follows: "An Influence Diagram is a graphical structure for modeling uncertain variables and decisions and explicitly revealing probabilistic dependencies and the flow of information." (7:871). The influence diagram has the advantage of graphically depicting a problem and how the variables are related; all in one picture. This is good not only from an analyst's view, but also for the decision-maker, who can verify that the correct problem is being addressed.

Recently, there has been much work by software developers to produce a viable package to solve influence diagrams. One such package is Decision Programming Language (DPL™). It gives the analyst a straightforward tool to model problems using influence diagrams and then solve them.

One of the limitations encountered using this software is the availability of only limited sensitivity analysis tools. The software includes an option to do deterministic one-way sensitivity analysis, but not any real two or more-way analysis. It is in this general area of sensitivity analysis that more work is needed. In this paper, Response Surface Methodology (RSM) is utilized as a sensitivity tool for the analysis of a problem solved with a influence diagram.

Research Objective

The objective of this research is to show that the use of response surface methodology can facilitate the construction of effective and efficient sensitivity analysis in the field of decision analysis. Effectiveness (to within an acceptable tolerance of error) is measured against current methods . Efficiency is evaluated through resource (analysis and computer time) utilization as compared to standard practice.

Benefits

There are a number of benefits from using response surface methodology for sensitivity analysis on a decision problem. The first is a substantial savings in the total number of model runs. As the model increases in complexity, the savings are tremendous. Subsequently, precious time to produce sensitivity information is salvaged. This time reduction is quite often the most significant from the decision-maker's point of view. The decision-maker receives the benefit of time-critical information as well as the added insight made available from the use of RSM. The additional information gained from the use of RSM is due to the extension of current analysis options, such as: the estimation of specific coefficients (effects) of significant variables, expanding the one-way sensitivity analysis to two-way and beyond, and the ability to track multiple factor interactions. The cost of these benefits is a limited loss of accuracy, well within the tolerances required for sensitivity analysis.

Scope

This thesis shows that RSM can be practically applied in the sensitivity analysis of a complex problem posed as an influence diagram. The measure of efficiency is subjectively evaluated, in a general comparison of RSM vs standard sensitivity analysis results. The sensitivity analysis is accomplished for both one-way and two-way perturbations of selected factors. This analysis is done to identify critical input variables and evaluate perturbations of their value, as well as perturbations of the their probability distributions, as they impact on the overall measurement of the solution. These input variables are subject to some uncertainty that is out of the control of the decision maker. The focus of the thesis is not to solve the problem being modeled but to evaluate the process itself. With this in mind, this paper presents an analysis which can be easily communicated to the decision maker without a need to understand all of the complex methods involved.

Assumptions

1. The response surface generated by the model can be accurately portrayed and analyzed using first or second order approximations over a small region of interest.
2. The primary focus of the decision-maker is to understand the underlying uncertainty involved in making the decision.

Results

This thesis shows that using RSM to do sensitivity analysis on problems solved with influence diagrams is a sensible approach. This methodology substantially increases the explanatory power of sensitivity analysis on decision problems. The approach utilized combines two proven analysis techniques to offer the analyst another tool and present the decision maker a more beneficial presentation. The results indicate that complex decision problems can be analyzed with a substantial savings in analyst man-hours and computer resources. The savings are a result of the utilization of RSM to estimate the response surface generated by solving the influence diagram. The subsequent predictive equations are highly accurate estimations, which are used to produce sensitivity information for various perturbations in the input variables.

Presentation of Results

Chapter II provides background on the fundamental tools in decision analysis. This includes information on decision trees, influence diagrams and solution capabilities. This chapter also includes the current framework of sensitivity analysis and some of the possibilities of the RSM metamodelling paradigm. This chapter builds the foundation for the methodology that is explained in Chapter III. Chapter III describes the model used, as well as providing a road map for the sensitivity analysis. This includes the standard sensitivity analysis offered by DPL™, several possible two-way sensitivity techniques, and the chronology of the response surface methodology. Next, an analysis of the results is presented in Chapter IV. The analysis includes the comparison of RSM to the other techniques used. Finally, Chapter V provides a summary of the conclusions, along with recommendations for further utilization of RSM in decision analysis.

II. Background

Decision Analysis

Howard and Matheson describe the job of the decision analyst "...as that of making 'opaque' situations 'transparent,' so that the [decision-maker] clearly sees what to do" (4:14). The overall goal of the decision analyst must be examined to evaluate the various tools of the trade. The analyst should construct a model that shows how all the variables are related and the outcomes of various decisions. It is critical to find the solution, but also to answer the correct question and clearly convey that answer to the decision maker. Clearly identifying the problem is an integral part of the analysis and is referred to as framing the problem. Framing is frequently the hardest part of the problem. The analyst has to identify the options available, the uncertain variables that relate to outcomes, and decide how to model the uncertain variables. Further, the relationship between these uncertain variables and the value must be determined. The analyst must select a tool that is capable of solving the problem and flexible enough to effectively communicate with the decision-maker.

Originally, the tool used most frequently to display and solve a decision problem was the decision tree (see Figure 2.1). The decision tree is useful for asymmetric problems with relatively few variables. This tool can be used to solve very complex problems, but these problems can turn the tree into a complex maze of branches with associated informational details. This disadvantage is not insurmountable for the analyst, but can be for the decision maker. The decision maker has to digest this large diagram and evaluate whether it portrays the decision at hand. Remember, the decision maker may know little about the analyst's tools. For this reason, influence diagrams have been used to portray the analyst's understanding of the problem. The calculations can be accomplished separately with the equivalent decision tree or influence diagram solver.

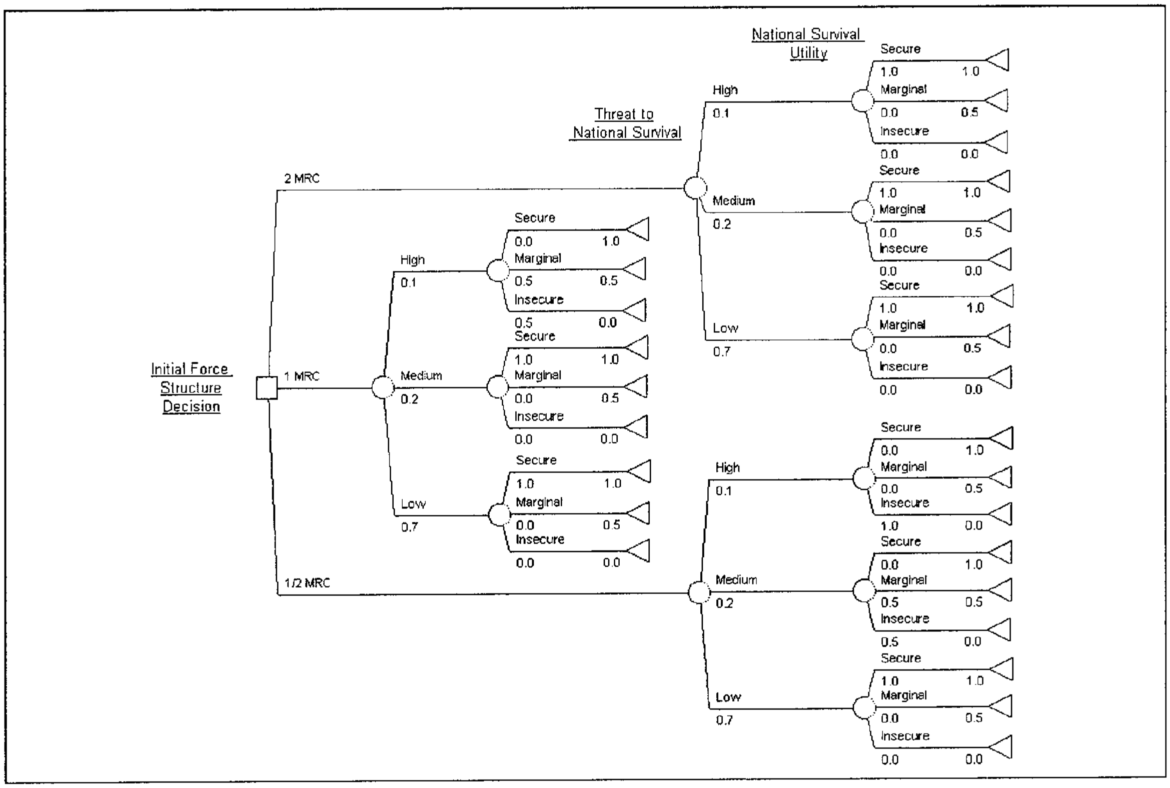


Figure 2.1 Decision Tree

Influence Diagrams. In an influence diagram, the problem situation is condensed to the decisions, the important uncertain variables, and the values (see Figure 2.2). These three node types are tied together with arcs to model the problem without all of the duplication of a decision tree. The goal is to use the influence diagram to communicate with the decision maker and use an underlying decision tree or influence diagram algorithm to do the analysis.

In 1986, Shachter proposed an algorithm for evaluating influence diagrams (7:871-872). This algorithm enabled the influence diagram to be solved directly. Shachter describes the influence diagram as, "... intuitive enough to communicate with decision makers and experts and, at the same time, precise enough for normative analysis" (7:871). The capability to solve the influence diagram was an important step for decision analysis. This capability provides analysts with a good communication tool in a convenient structure for computer solution techniques (8:603).

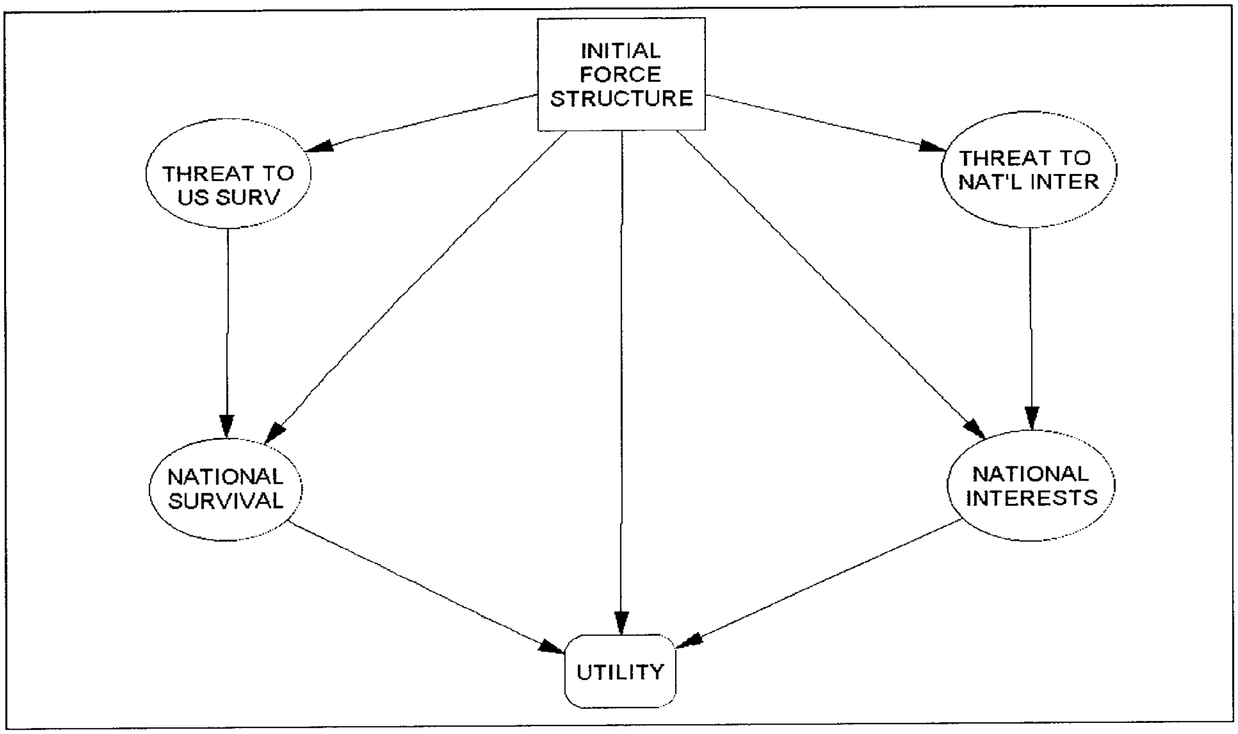


Figure 2.2 Influence Diagram

However, this technique has not been used much outside of academic circles until recently. Decision Programming Language (DPL^{TM}) has changed that with the ability to solve influence diagrams via a low cost microcomputer software package. DPL^{TM} solves the influence diagram by using an influence diagram algorithm to do the calculations (1). This allows it to accomplish deterministic sensitivity analysis on one variable at a time. The output includes cumulative risk profiles of the alternatives, rainbow diagrams, and tornado diagrams. A general limitation of the technique (and the software) is the inability to do in-depth (two-way or higher) sensitivity analysis.

Sensitivity Analysis

Although the sensitivity of any one variable can be an effective screening tool for many problems, stopping here would leave out a large part of the potential information gained from a model. If the problem contains many variables, what is the effect of two or three being slightly different than anticipated? If the answer doesn't change the recommended action, it doesn't matter. If it does change the recommendation, the decision maker needs to know. One feasible approach would be to exhaustively enumerate the possibilities and run the model at all the possible combinations of variable settings. This is reasonable for small problems with few variables, but becomes a exponentially large process with increased problem size. An approach that significantly reduced the computational complexity was suggested by Smallwood and Morris (9:61-80). Smallwood and Morris used one-way sensitivity analysis to screen the critical random variables from the rest of the field. They realized that to run a full factorial design of the nine variables at three levels would take 3^9 or 19683 individual runs of the model. This would take up a costly amount of analyst time and computer resources, so they decided to group the nine critical state variables into three classes. They developed a worst case scenario for each class of variables which defined eight global scenarios (9:61-80). Running these global scenarios proved to be a effective analysis tool for generating strategy regions, but condensing the variables requires general assumptions about the groupings and combined effects. This technique provides additional information to the decision maker, but limits the ability to trace variable effects beyond the variable group.

Response Surface Methodology

A different approach was used by Stone, who studied the non-conformity of parts in the Air Force Supply System (10). A simple model was developed for determining the best policy for sampling parts in the inventory. The model was displayed as an influence diagram and solved using INDIA. The sensitivity analysis was done using response surface methodology. Stone found that " RSM [Response surface methodology] provides for a better sensitivity analysis because RSM explores the interaction effects between the variables in a more structured, consistent, and reliable way" (10:2). Stone's thesis provides initial groundwork for applying response surface methodology to decision analysis problems.

Complex models are frequently used in the field of simulation. Large Air Force simulation models take hours to make one run, so common techniques like multiple-run sensitivity analysis are difficult to perform. Response surface methodology has been used to do sensitivity analysis on many of these simulation models. These models have many variables and usually only a fraction of them are critical over a small design region. Meidt and Bauer take advantage of this property and use response surface methodology to create metamodels for simulation (see Figure 2.3) (6:183-191).

In their article, Meidt and Bauer describe the process of group screening as follows: "Group screening combines sets of factors that are believed to exhibit like behavior in the model. Each group is treated as a single variable and regressed against the responses using a low resolution design" (6:184). Group screening is similar to single factor screening in that it can be accomplished with a low resolution design to narrow the field. This technique would be extremely beneficial where the uncertain variables are closely correlated to the response in a large model. It would drastically reduce the number of runs required while validating the lumping of variables where the groups proved to be insignificant. Such reduction would have an application in the sensitivity analysis of many complex decision analysis problems.

Once the group screening narrows the field to critical groups of input variables, another low resolution design (resolution III) is run on the main factors remaining. From here, a first order (linear) model can be used to estimate the response surface over a small area of interest. This model is regressed and tested for lack of fit. If the model proves to

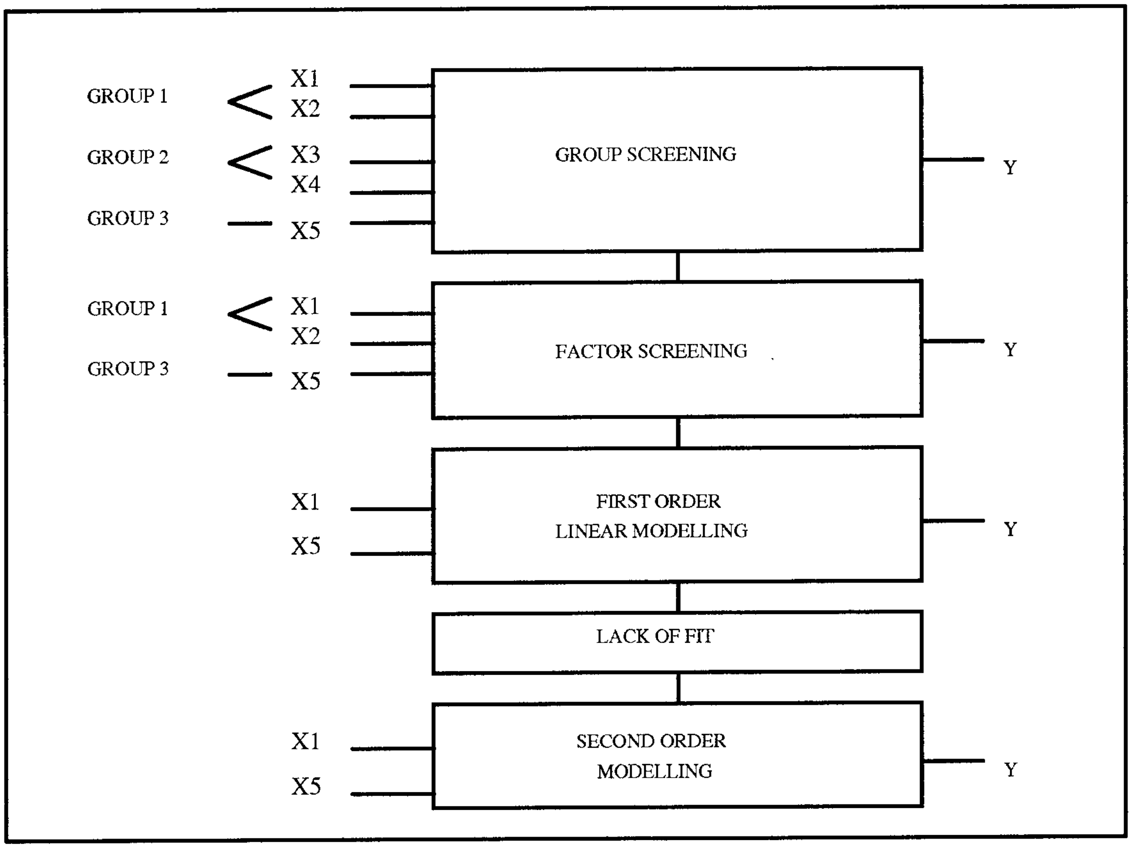


Figure 2.3 RSM Metamodelling Paradigm

*Adapted from (6:184)

be significant, then the method of steepest ascent can be used as a gradient search technique. At some point, there may be a lack of fit and a second order model can be used to better fit the surface and eventually locate local optimum. This second order model requires a higher resolution (resolution V or better) design to include the interaction terms. Using this methodology, valuable insight is gained into the response function over a range of inputs. This technique can not only be used for optimization, but also for sensitivity analysis on the predicted surface.

III. Methodology

Problem Description

Air Force decision makers are greatly interested in expanding the ability to evaluate uncertainty in modeling. This ability requires the analysis of uncertainty in the model parameters, including subjective value and probability inputs, and presenting the results in a understandable format.

Here, a decision analysis model is developed to examine the major uncertainties in assessing force requirements. The DOD is significantly restructuring U.S. military forces. Until recently, the forces were designed to fight and win two major regional contingencies (MRC). Due to significant budget constraints, the U.S. must consider military force structures with lower capabilities. However, significant uncertainties exist about future threats to national survival and national interests. Suppose there are three force structure alternatives under consideration: two MRC force, one MRC force, and one-half MRC force. The threat to national survival and national interests over the next 20 years can be grouped into three categories: high threat, medium threat, and low threat. National interests include threats to countries that have international security agreements with the U.S. Suppose the following probabilities are obtained from intelligence estimates:

Threat	Threat to National Survival	Threat to National Interests
High	0.1	0.3
Medium	0.2	0.6
Low	0.7	0.1

Table 3.1 Probability of Threats to National Survival and National Interests

Next, the intelligence data on the expected threat is included. The predictions are based on the assumption that the intelligence community has a higher probability of correctly predicting threats to national survival than to national interests. The following probabilities are for the intelligence prediction given the actual threat to national survival:

Intelligence Prediction \ Threat to National Survival	High	Medium	Low
High	0.8	0.2	0.0
Medium	0.2	0.6	0.4
Low	0.0	0.2	0.6

Table 3.2 Intelligence Prediction given Threat to National Survival

The following probabilities are for the intelligence prediction given the actual threat to national interests:

Intelligence Prediction \ Threat to National Interests	High	Medium	Low
High	0.7	0.3	0.2
Medium	0.2	0.4	0.3
Low	0.1	0.3	0.5

Table 3.3 Intelligence Prediction given Threat to National Interests

National objectives are to insure national survival and to defend national interests. The possible outcomes of National Survival and National Interests are:

- Secure
- Marginal
- Insecure

The probabilities of the outcomes of National Survival and National Interests depend on the force structure decision and the threat. The following table provides the outcomes and probabilities:

Force Structure	Threat	National Survival / National Interests
Two MRC	All	Secure, 1.0
One MRC	High	Marginal, 0.5 Insecure, 0.5
One MRC	Medium, Low	Secure, 1.0
One-half MRC	High	Insecure, 1.0
One-half MRC	Medium	Marginal, 0.5 Insecure, 0.5
One-half MRC	Low	Secure, 1.0

Table 3.4 Probability of outcomes given Force Structure and Threat

There are two decisions to make; the first is the initial force structure, and when the threat is updated, the level of force buildup. The force buildup has the same options (1/2, 1, or 2 MRC) given sufficient lead time. The value of possible outcomes is measured with an additive utility function consisting of three individual utilities. These individual utilities are weighted according to national objectives. The overall utility function looks like this:

$$U(\text{Survival}, \text{Interests}, \text{Cost}) = a * U(\text{Survival}) + b * U(\text{Interests}) + c * U(\text{Cost}) \tag{3-1}$$

The weights of the three utilities must sum to one to meet the independence criteria. The utilities are assigned so that a value of one is the best outcome. Hence, we are striving to maximize utility. The individual utilities range from zero to one and are assigned according to the outcomes shown in Tables 3.5 and 3.6.

Outcome	National Survival Utility	National Interests Utility
Secure	1.0	1.0
Marginal	0.5	0.5
Insecure	0.0	0.0

Table 3.5 Utility of National Survival and National Interests given Outcome

Initial Force Structure	Force Structure Buildup	Force Structure Cost Utility
Two MRC	No Change	0.0
One MRC	Two MRC	0.2
One MRC	No Change	0.4
One-half MRC	Two MRC	0.5
One-half MRC	One MRC	0.7
One-half MRC	No Change	1.0

Table 3.6 Force Structure Cost Utility given Initial and Buildup Force Structure

Unfortunately, if we decide to build up our force structure, the buildup may not be complete in time to meet the threat. Therefore, this uncertainty is modeled with a force available variable. The assumed probabilities are found in Table 3.7.

Initial Force Structure	Force Structure Buildup	Two MRC	One MRC	One-half MRC
Two MRC	No Change	1.0	0.0	0.0
One MRC	Two MRC	0.5	0.5	0.0
One MRC	No Change	0.0	1.0	0.0
One-half MRC	Two MRC	0.125	0.375	0.5
One-half MRC	One MRC	0.0	0.5	0.5
One-half MRC	No Change	0.0	0.0	1.0

Table 3.7 Probability that the Force Structure is available to meet Threat

Model Description

Understanding the Influence Diagram. The objective of this model is to optimize the utility of the force structure decision. The initial force structure decision includes the three force structure alternatives previously discussed, but are not displayed in the influence diagram. This is where the difference between a decision tree and a influence diagram becomes apparent. The influence diagram depicts a macroscopic view of the problem with emphasis on how the variables are related. What is not seen is the underlying branches of alternatives and probabilities that have been defined in the framing process. This high-resolution information is hidden, while the critical relationships among the variables are shown as arcs. These arcs define when information is known relative to decisions and whether variables are probabilisticly dependent on one another.

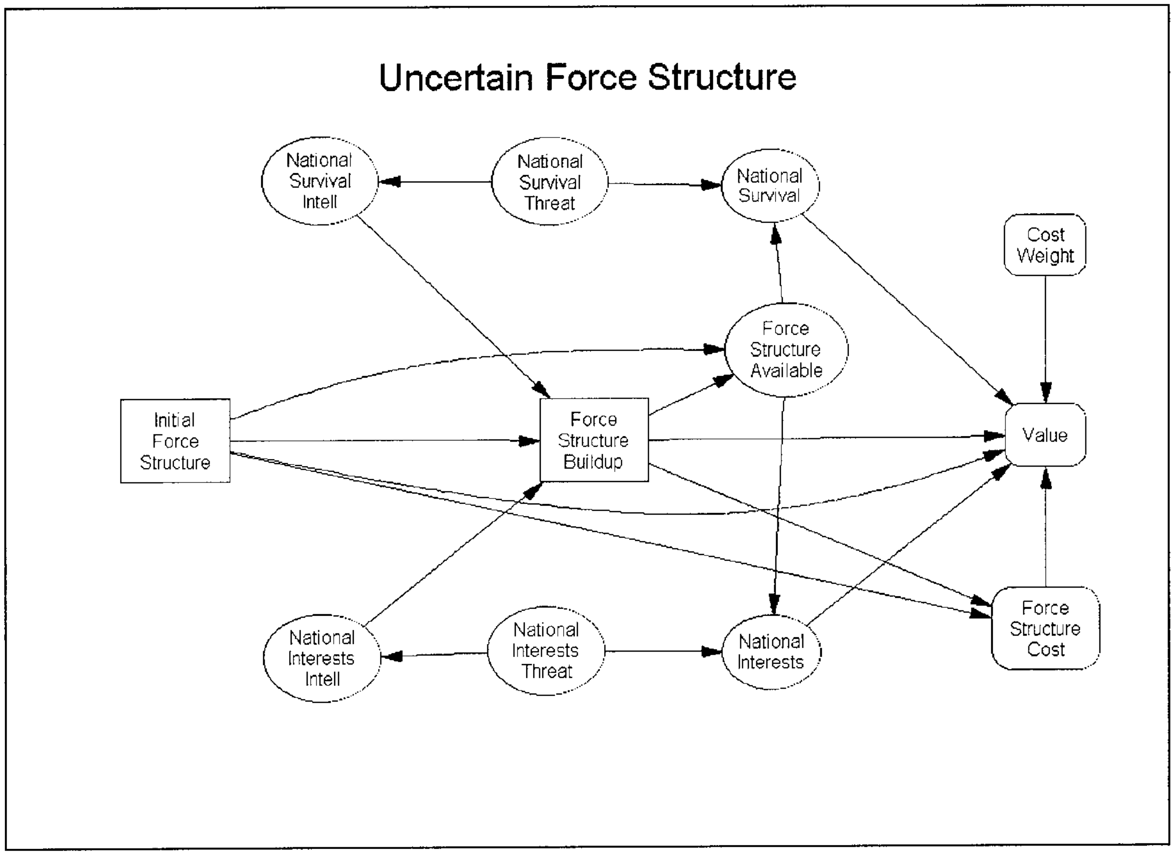


Figure 3.1 Influence Diagram of Force Structure Decisions

The influence diagram in Figure 3.1 shows that the initial force structure decision will affect the force structure buildup decision, the force structure available, and the force structure cost. It is sensible that the initial force structure will determine the options for a force structure buildup, as well as the overall cost of the force. The force structure cost, defined with a utility ranging from zero to one, is determined by the force structure chosen and is represented with a value node. The value node has no probabilistic branches. The value node information is deterministic once the inputs are defined. This node's information is then used as an input to our overall utility function contained in the final value node. These relationships of one node to another outline the fundamental structure of the model. This structure is simply explained in the influence diagram providing a powerful communication tool.

Input Variables. The model contains seven uncertain variables displayed as ovals in the influence diagram. These uncertain variables are derived from the tables found previously in the problem description. These variables are listed below:

National Survival threat **Intelligence** prediction - the probability of intelligence predicting future national survival threats given the actual threat.

National Survival Threat - the probability of the actual national survival threat being either high, medium, or low.

National Survival - the US national survival probability and utility of being secure, marginally secure, or insecure dependent on the actual national survival threat and force structure available.

Force Structure Available - the probability of the force structure attaining the desired buildup level given the initial force structure.

National Interests threat **Intelligence** prediction - the probability of intelligence predicting future national interests threats given the actual threat.

National Interests Threat - the probability of the actual national interests threat being either high, medium, or low.

National Interests - the US national interests probability and utility of being secure, marginally secure, or insecure dependent on the actual national interests threat and force structure available.

The remaining nodes are for the force structure cost, the overall utility function, and the cost weight parameter. The force structure cost node identifies the utility of all the possible combinations of initial force structure and force structure changes. The overall utility function is the sum of the three individual utilities weighted by the cost weight parameter as seen in Equation (3-2).

$$U(\text{Survival}, \text{Interests}, \text{Cost}) = 0.5 * U(\text{Survival}) + 0.25 * U(\text{Interests}) + 0.25 * U(\text{Cost}) \quad (3\text{-}2)$$

This cost weight was initially set to assert that the national survival utility is twice as important as the national interests or force structure cost utilities.

Output Variables. The model's one output is the maximum expected utility of the optimal decisions. This includes the expected utility ranging from zero to one, but also the decision policy that would yield this optimal number. There are six possible decision policies including the force structure buildup options. The optimal expected utility is a maximum of the three initial force structure alternative's expected utilities. This fact can be used to break the output down into three separate responses. These three response surfaces are evaluated and discussed in the RSM sensitivity analysis section.

Sensitivity Analysis using DPL™

Grouping Variables and Initial Perturbances. Initially the variables were placed into three main groups. All of the variables which were probabilities, such as the probability of the threat to national survival being high, were grouped as probability variables. The probability variable group included 51 individual variables. The utility variable group included 12 variables, such as the force structure cost utility of a one MRC initial force with a buildup to a two MRC force. The last group is the utility weight group, which includes the three weighted coefficients in the overall utility function. The perturbations of these various groups are limited by the relationships in each.

The probabilities of the various outcomes must obviously sum to one and therefore had to be perturbed in much the same way as the utility weights. This vector of probabilities is varied by changing the main probability, and altering the other two elements of the set by a negative one-half the change. The individual main probabilities are all varied by plus or minus 0.1 about their initial value. An example of how the National Survival threat probabilities are perturbed is shown in Table 3.8.

Threat	Threat to National Survival	Perturbation of probability vector
High	0.1	0.1 + ProbabilityPerturbation
Medium	0.2	0.2 - (0.5*ProbabilityPerturbation)
Low	0.7	0.7 - (0.5*ProbabilityPerturbation)

Table 3.8 Perturbation of National Survival Threat Probability (NSTP)

The utility variable group contains the various utility weights which are assessed by the decision maker for each possible outcome or value. The best utility is subjectively set to one with the worse being zero. The best outcome of National Survival utility would be secure, while the best force structure cost utility would be the cheapest or the one-half MRC force, and both are assigned a utility of one. Here, the perturbations only affect the outcomes which fall between the best and worse case. This is due to how one and zero are assigned. These central utilities were varied by plus or minus 0.1, with all the central outcomes being either increased or decreased at the same time. An example of how Force Structure Cost Utility is perturbed is shown in Table 3.9.

The utility weight coefficients must sum to one in order to meet the additive utility function requirements. This leads to a forced grouping where perturbing one coefficient

Initial Force Structure	Force Structure Buildup	Force Structure Cost Utility	Perturbation of Force Structure Cost Utility
Two MRC	No Change	0.0	0.0
One MRC	Two MRC	0.2	0.2 +/- UtilityPerturbation
One MRC	No Change	0.4	0.4 +/- UtilityPerturbation
One-half MRC	Two MRC	0.5	0.5 +/- UtilityPerturbation
One-half MRC	One MRC	0.7	0.7 +/- UtilityPerturbation
One-half MRC	No Change	1.0	1.0

Table 3.9 Perturbation of Force Structure Cost Utility (FSCU)

requires a change to the others. The original utility function uses the fact that National Survival Utility is twice as important as either National Interests or Force Structure Cost Utilities. This dependence is expressed with the following function:

$$U(\text{Survival}, \text{Interests}, \text{Cost}) = (1-2C) * U(\text{Survival}) + C * U(\text{Interests}) + C * U(\text{Cost}) \qquad (3\text{-}3)$$

The coefficient C is initially set to 0.25.

The influence diagram of the original sensitivity analysis on the three variable groups is shown in Figure 3.2. There are three value nodes used as perturbation variables: Probability Perturbation, Utility Perturbation, and the Cost Weight perturbation. These are the variables which are actually changed across the ranges previously defined. The other variables are defined in terms of their particular perturbation variable. This Influence Diagram is used to generate the tornado diagram found in Figure 3.3.

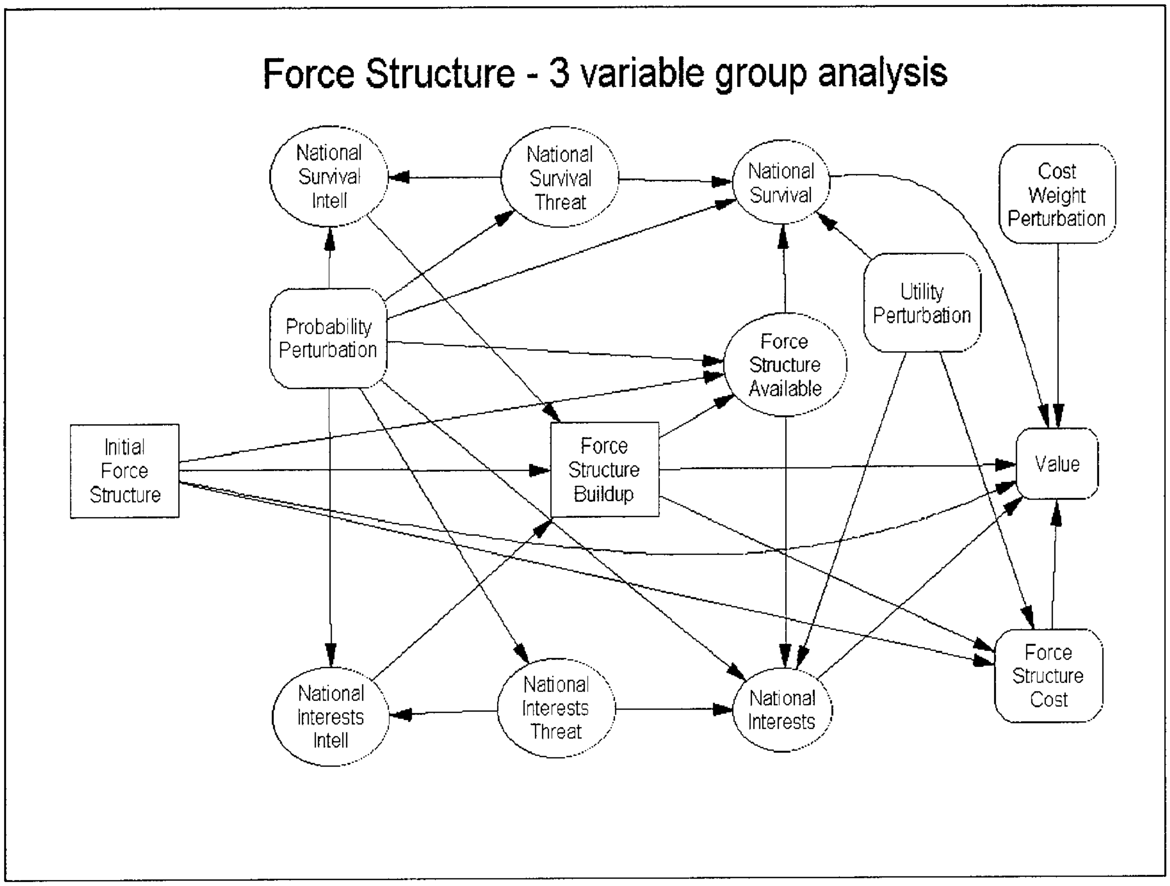


Figure 3.2 Influence Diagram for Three Group Perturbations

The tornado diagram presents the effects of perturbing one main group at a time. This one-way sensitivity analysis displays the effect of the perturbations on the overall expected utility and any changes in decision policy. The shaded areas indicate that the optimal alternative has changed from the original setting. DPL™ evaluates the model at three points; the original base case and two endpoints on either side of the original. The shaded area shows only that the change in policy occurred somewhere between the original and the endpoint evaluation.

This initial group analysis gives some insight into the effect of changing the probability, utility or utility weight parameters. Perturbing any of these parameters can lead to a change in expected utility and/or decision policy. The objective so far is to gain

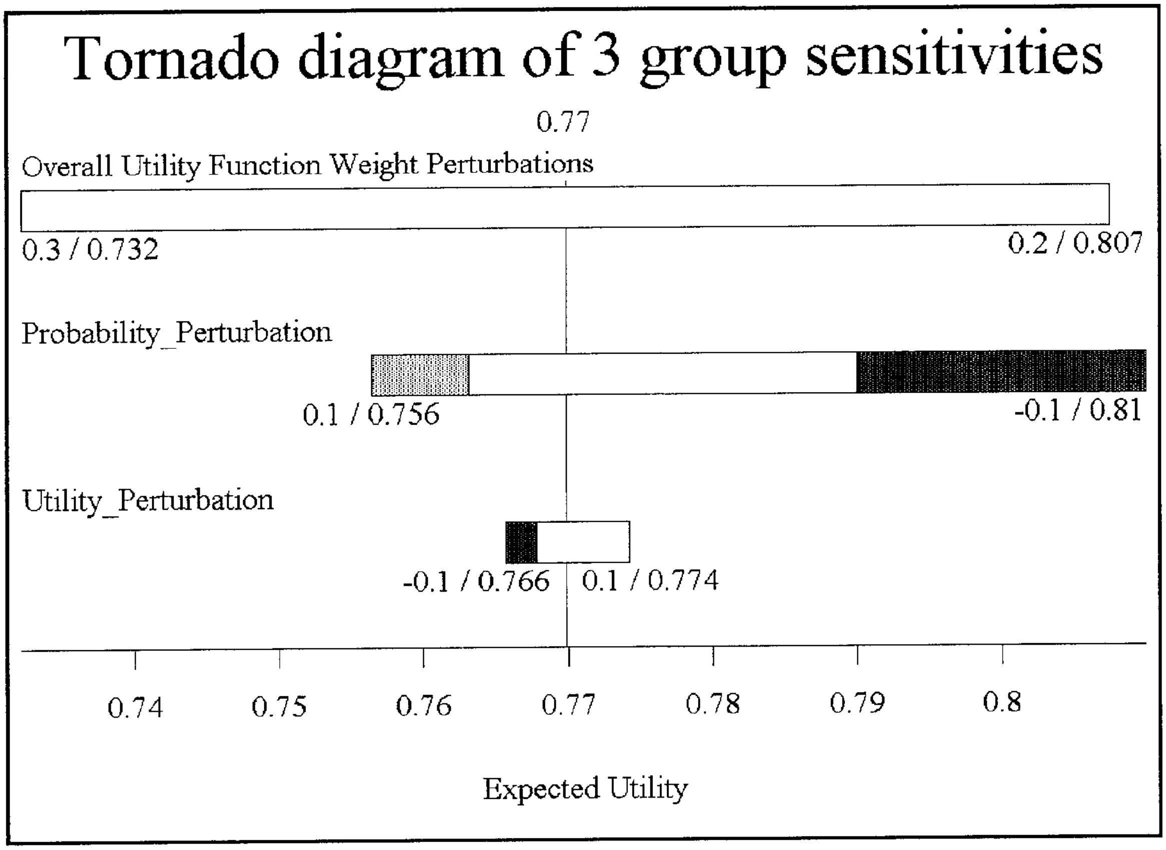


Figure 3.3 Tornado Diagram of Three Main Groups One-way Sensitivity Analysis

insight into the significant variables. From here, it would be useful to narrow the field to study the most important or sensitive variables. At this point, there is no evidence to suggest throwing any of the groups out. Therefore, the number of groups was expanded by dividing them into 11 subgroups

Grouping at a Lower Level of Detail. The eleven new groups were defined by using the preexisting nodes in the influence diagram. The probability group was broken down according to the uncertain variables in the chance nodes. The utility group was broken down into the three smaller utility groups; national survival utility, national interests utility and force structure cost utility. The overall weight coefficient group could not be looked at in any finer detail, so it remained unchanged.

An influence diagram with value nodes identifying the eleven group perturbations is shown in Figure 3.4.

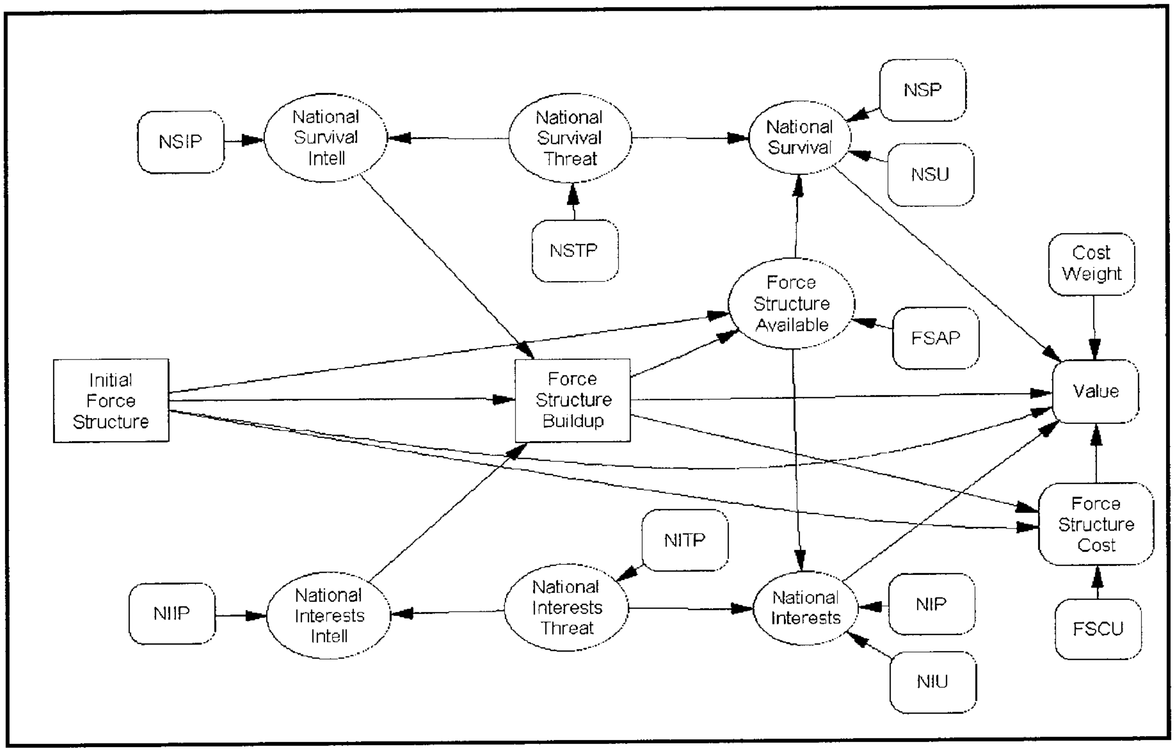


Figure 3.4 Influence Diagram 11 groups used for Sensitivity Analysis

Group Variables used to make perturbations for sensitivity analysis:

National Survival threat **Intelligence** prediction **Probabilities** - NSIP

National Survival Threat Probabilities - NSTP

National Survival Probabilities - NSP

National Survival Utility - NSU

Force Structure Available Probabilities - FSAP

Cost Weight coefficient to overall utility function - CW

Force Structure Cost Utility - FSCU

National Interests threat **Intelligence** prediction **Probabilities -** NIIP

National Interests Threat Probabilities - NITP

National Interests Probabilities - NIP

National Interests Utility - NIU

One-way Sensitivity Analysis. These groups were perturbed similar to the previous groupings to create a tornado diagram shown in Appendix E. The probability and utility perturbations were all plus and minus 0.1 from the significant element in each vector. The cost weight was again varied from 0.2 to 0.3.

The top four groups have the greatest effect on expected utility for the range of values selected. Seven of the top eight indicate at least one change in decision policy at some point in the perturbation range. Remember, this figure is based on changing one parameter at a time with the others set at their base value. This gives no information on how the decision is affected if more than one group is varied.

Two-way Sensitivity Analysis. DPL's rainbow diagram option is used to do two-way sensitivity analysis. The rainbow diagram runs the model across a range of settings (up to 21) for one variable. If there is a specified relationship between two variables, they can be varied at the same time by defining one variable in terms of the other, or by defining both variables in terms of a third. The other possibility is to define an array with settings of the two variables and running the model at each setting in the array (1:333-357). The best option for this model was to use a rainbow diagram of one variable across the interval -0.1 to 0.1 and run this rainbow diagram eleven times. The second variable was incremented between runs to establish a strategy region graph seen in Table 3.10. The extraction of the data from the rainbow diagrams after each run was a rather painstaking process, but it did provide the means to gather two-way sensitivity information.

The strategy region displays the optimal decision policy across the desired range of interest. The line that splits the two decision strategies does not lie on the boundary of values. This line may lie anywhere between the points evaluated in the 0.02 wide interval. This level of accuracy is acceptable in most cases, since the strategy region will show the general trend.

		FSCU										
		-0.1	-0.08	-0.06	-0.04	-0.02	0	0.02	0.04	0.06	0.08	0.1
	-0.1	1	1	1	1	1	1	1	1	1	1	1
	-0.08	1	1	1	1	1	1	1	1	1	1	1
	-0.06	1	1	1	1	1	1	1	1	1	1	1
	-0.04	1	1	1	1	1	1	1	1	1	1	1
	-0.02	2	1	1	1	1	1	1	1	1	1	1
NSTP	0	2	2	1	1	1	1	1	1	1	1	1
	0.02	2	2	1	1	1	1	1	1	1	1	1
	0.04	2	2	2	1	1	1	1	1	1	1	1
	0.06	2	2	2	2	1	1	1	1	1	1	1
	0.08	2	2	2	2	2	1	1	1	1	1	1
	0.1	2	2	2	2	2	2	1	1	1	1	1

Table 3.10 Decision Policy Based on Perturbations of Force Structure Cost Utility vs National Survival Threat Probability (1 or 2 MRC Force)

If the estimation of the most likely values of the uncertain variables is near the break point between a one and two MRC force, the optimal decision would be highly sensitive to the actual outcome of these variables. It is critical to realize that these figures are based on expected outcomes and must not be taken literally. In other words, as the estimates approach the bottom left corner of Table 3.10, the level of confidence increases that a two MRC force is the best decision. Another approach that would give the same results would be to use the actual equations generated from folding back the decision tree (3:121-136). This would be a ambitious undertaking, considering that the equation for each of the three alternatives would require 29,439 multiplication's, 19,626 additions, and 81 comparisons. This is based on a symmetric decision tree that would be an expanded version of Figure 3.5. These equations would have to be entered manually into a spreadsheet and compared at the various input values of interest. This approach would certainly save on the number of model runs, but would be impractical for all but the simplest of models.

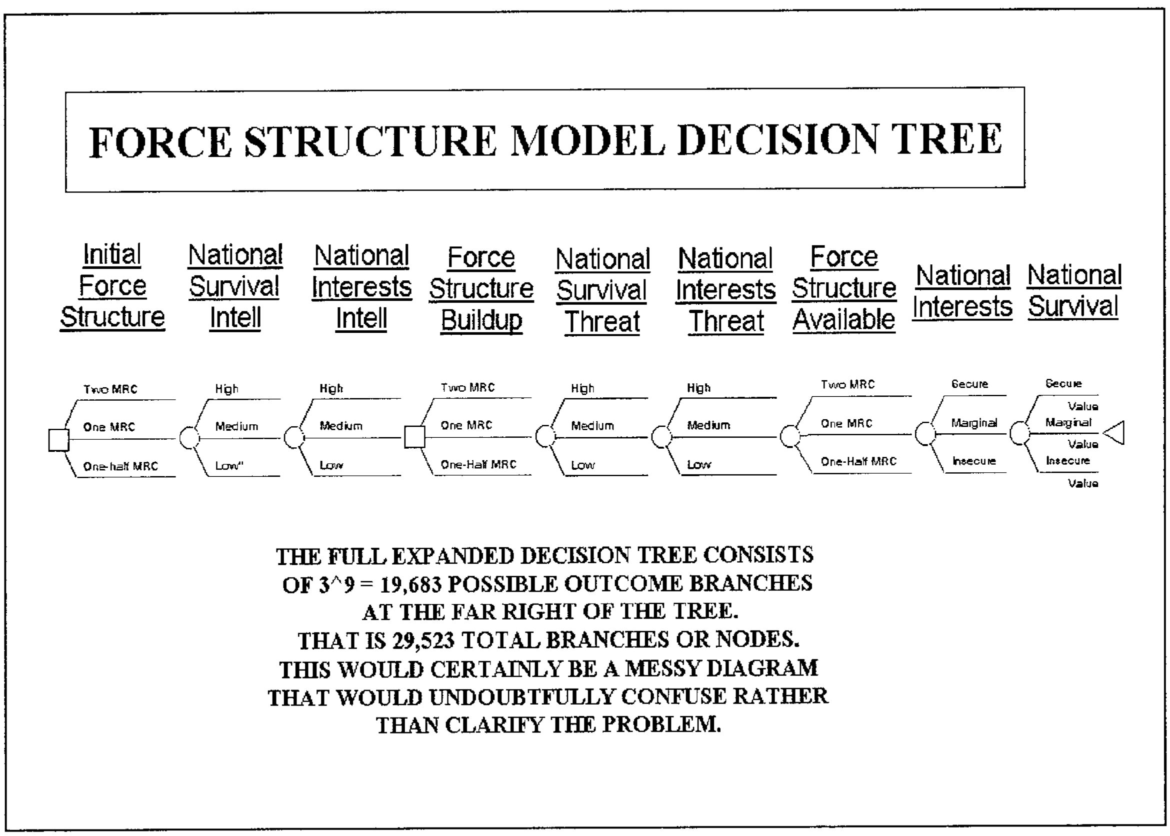


Figure 3.5 Condensed Decision Tree for Force Structure Problem

Although the one-way sensitivity analysis of this decision model is fairly straightforward, the ability to perform higher level analysis is quickly restricted by a limited number of tools in the software. This is where there is room for improvement.

Response Surface Methodology Approach to Sensitivity Analysis

Response Surface Methodology (RSM) is simply a method of estimating a response surface through experimental design. The end result being a parsimonious model that closely predicts the results of the model in some region of interest. This would certainly aid in doing sensitivity analysis on the model if we could accurately portray the response surface with an equation that doesn't include almost 50,000 arithmetic operators.

To begin this portion of the analysis, the eleven groups described previously were used. Initially, to determine if this would work at all, a Plackett-Burman design was used to screen these eleven groups for significant factors. This required a 12 run design that was of resolution III (2:162). Therefore, the eleven main factors would not be confounded with main factors, but any interactions would be confounded with the main factors. The main factors were all coded to a range of 1 to -1 to ease the programming and analysis. This coding was done using the standard methods shown in Equation (3-4).

$$X_{CODED} = (X_{UNCODED} - \delta_{MIDPOINT}) / \lambda_{HALF-WIDTH} \qquad (3\text{-}4)$$

where $\delta_{MIDPOINT}$ = the midpoint of the interval of perturbation (0 or 0.25)

$\lambda_{HALF\text{-}WIDTH}$ = the half-width of the interval of perturbation (0.1 or 0.05)

Therefore, an $X_{UNCODED}$ = -0.1 would transform to a X_{CODED} = -1.0. The 12 runs were made using the uncoded values in DPL™. The coded values from the design matrix were regressed against the expected utility response Y.

The original Plackett-Burman design matrix and response is shown in Table 3.11.

GROUP FACTOR		NSIP	NIIP	NSTP	NITP	NSP	NSU	NIP	NIU	FSAP	FSCU	CW	E(UTIL)
RUN	I	1	2	3	4	5	6	7	8	9	10	11	Y
1	1	1	1	-1	1	1	1	-1	-1	-1	1	-1	0.835704
2	1	-1	1	1	-1	1	1	1	-1	-1	-1	1	0.700124
3	1	1	-1	1	1	-1	1	1	1	-1	-1	-1	0.8
4	1	-1	1	-1	1	1	-1	1	1	1	-1	-1	0.814776
5	1	-1	-1	1	-1	1	1	-1	1	1	1	-1	0.825202
6	1	-1	-1	-1	1	-1	1	1	-1	1	1	1	0.76432
7	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	0.81264
8	1	1	1	-1	-1	-1	1	-1	1	1	-1	1	0.745588
9	1	1	1	1	-1	-1	-1	1	-1	1	1	-1	0.821544
10	1	-1	1	1	1	-1	-1	-1	1	-1	1	1	0.711047
11	1	1	-1	1	1	1	-1	-1	-1	1	-1	1	0.7
12	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0.8264

Table 3.11 Initial Plackett-Burman Design on 11 Group Factors

The resulting ANOVA table suggests that the only significant factors are the National Survival Threat Probability (NSTP), Force Structure Cost Utility (FSCU), and the Cost Weight (CW). Also, by examining the intercept, which is 0.776614 to the expected value of 0.769821 of the base case, it seems that the regression line will hold across the entire range. This suggests that a linear first order model might be sufficient to approximate the response surface. After a number of variations, a 2^3 full factorial design with the center point included was run on the three significant factors. The equation that surfaced is listed below with the accompanying ANOVA Table 3.12.

$$E(U) = 0.776614 - 0.01629 * NSTP + 0.018088 * FSCU - 0.04031 * CW$$
$$-0.00691 * NSTP * FSCU + 0.00781 FSCU * CW \qquad (3\text{-}5)$$

Regression Statistics		Analysis of Variance					
Multiple R	0.996737		df	SS	MS	F	Signif F
R Square	0.993484	Regress	5	0.01861	0.003722	91.47956	0.001775
Adjusted R Square	0.982624	Residual	3	0.00012	4.07E-05		
Standard Error	0.006379	Total	8	0.01873			
Observations	9						

	Coef	Std Err	t Statistic	P-value
Intercept	0.776614	0.002126	365.2378	3.54E-18
NSTP	-0.01629	0.002255	-7.22163	9.05E-05
FSCU	0.018088	0.002255	8.020193	4.29E-05
CW	-0.04031	0.002255	-17.8756	9.83E-08
NSTP*FSCU	-0.00691	0.002255	-3.06477	0.01547
FSCU*CW	0.00781	0.002255	3.462943	0.008531

Table 3.12 Analysis of Variance Table for regression of:

E(U)=b0+b1*NSTP+b2*FSCU+b3*CW+b4*NSTP*FSCU+b5*FSCU*CW

The equation turns out to be quite accurate with a highly significant F-test and a R square and adjusted R square which explains much of the data. The individual factor t-tests also show a high level of confidence with a p-value below 0.02 in all cases. This first success at predicting the response surface of the force model leads to further exploration of it's possible applications. At this point, there is an accurate approximation of the optimal expected value across the region of interest. This is useful for the prediction of the optimal expected value, but it doesn't define any changes in policy. Without the ability to predict changes in the optimal policy, the results are not very useful.

The next step is to develop an equation for each of the three alternatives and use these to produce strategy region graphs. With this in mind, the initial Plackett-Burman design runs were used with the addition of the expected Utility of each of the three alternatives. The first regression on the response for the two MRC alternative was quite surprising. The only significant factor was that of Cost Weight. After a little refining of the equation, it turns out that we can perfectly predict the expected utility of the two MRC option. The expected value of this option can be defined with Equation (3-6).

$$\mathbf{E(U)_{2MRC} = 0.75 - 0.05 * CW} \qquad (3\text{-}6)$$

Remember that these equations are for the coded values of the input variables. The coded value CW is a function of the uncoded cw and is determined by Equation (3-7).

$$\mathrm{CW} = (\mathrm{cw} - 0.25) / 0.05 \qquad (3\text{-}7)$$

Therefore a value of 0.2 would be coded as -1 in the design matrix and so on. It is a simple matter to reverse the process to decode these variables and use the regression equations.

The same basic procedure was used to identify an equation for the expected utility of the 1 MRC alternative. However, the initial Plackett-Burman design suggested that there were four groups that were significant to approximate this response. These significant groups were NSTP, NITP, FSCU and CW. In order to accurately examine these factors a 2^4 full factorial design was run using all four main factors and all combinations of interaction terms. The follow-on analysis of the ANOVA table culled the field to a linear combination of the four factors with one interaction term. The final approximating Equation (3-8) and the associated ANOVA Table 3.13 are found below.

$$E(U)_{1MRC} = 0.771733 - 0.02365 * NSTP - 0.0156 * NITP + 0.025 * FSCU \\ -0.03726 * CW + 0.0058 * FSCU * CW \quad (3\text{-}8)$$

Regression Statistics			ANOVA					
				df	SS	MS	F	Sig F
Multiple R	0.995456							
R Square	0.990932		Regression	5	0.045458	0.009092	240.4038	7.6E-11
Adjusted R Square	0.98681		Residual	11	0.000416	3.78E-05		
Standard Error	0.00615		Total	16	0.045874			
Observations	17							

	Coef	Std Error	t Stat	P-value
Intercept	0.771733	0.001492	517.4185	1.76E-25
NSTP	-0.02365	0.001537	-15.3816	8.75E-09
NITP	-0.0156	0.001537	-10.1473	6.39E-07
FSCU	0.025	0.001537	16.26113	4.86E-09
CW	-0.03726	0.001537	-24.2375	6.74E-11
FSCU*CW	0.005	0.001537	3.252225	0.007705

Table 3.13 Analysis of Variance Table for Regression of 1 MRC option: E(U)=b0+b1*NSTP+b2*NITP+b3*FSCU+b4*CW+b5*FSCU*CW

The results show that we can explain the data at a high degree of confidence with residuals limited to the third or fourth decimal place. Although this equation is a bit more complex than that of the two MRC option, it is still easily applied to spreadsheet analysis and the development of strategy regions. Here again the group factors agree with the overall tornado diagram as to the level of significance and relative ranking. The next step is to look at the last alternative of a one-half MRC force.

The one-half MRC force response was similarly examined against the Plackett-Burman design. In this case there was evidence that three group factors were significant. These significant group factors were NSTP, FSAP, and CW. These factors were used to create a 2^3 full factorial design, which was run using the DPL™ model. The resulting approximation Equation (3-8) and ANOVA Table 3.14 are shown below.

$$E(U)_{1/2MRC} = 0.724785 - 0.03329 * NSTP - 0.02035 * FSAP - 0.02033 * CW + 0.006069 * NSTP * CW \quad (3\text{-}9)$$

Regression Statistics	
Multiple R	0.999163
R Square	0.998326
Adjusted R Square	0.996651
Standard Error	0.002627
Observations	9

ANOVA

	df	**SS**	**MS**	**F**	**Sig F**
Regression	4	0.016463	0.004116	596.2669	8.4E-06
Residual	4	2.76E-05	6.9E-06		
Total	8	0.016491			

	Coef	**Std Error**	**t Stat**	**P-value**
Intercept	0.724785	0.000876	827.6086	1.28E-11
NSTP	-0.03329	0.000929	-35.8392	3.62E-06
FSAP	-0.02035	0.000929	-21.905	2.57E-05
CW	-0.02233	0.000929	-24.0438	1.77E-05
NSTP*CW	0.006069	0.000929	6.533262	0.002836

Table 3.14 Analysis of Variance for Regression of 1/2 MRC:

E(U)=b0+b1*NSTP+b2*FSAP+b3*CW+b4*NSTP*CW

This equation is also quite significant with good explanatory power. Although this alternative does not become a optimal choice in the region of interest, this equation can still be a useful tool for sensitivity analysis.

Using Response Surface Approximations to Generate Strategy Regions. The strategy region graph presented in the standard sensitivity analysis section is a excellent tool for presenting two-way analysis. The new approach is to use the three equations generated through the use of RSM to estimate each alternative's expected utility.

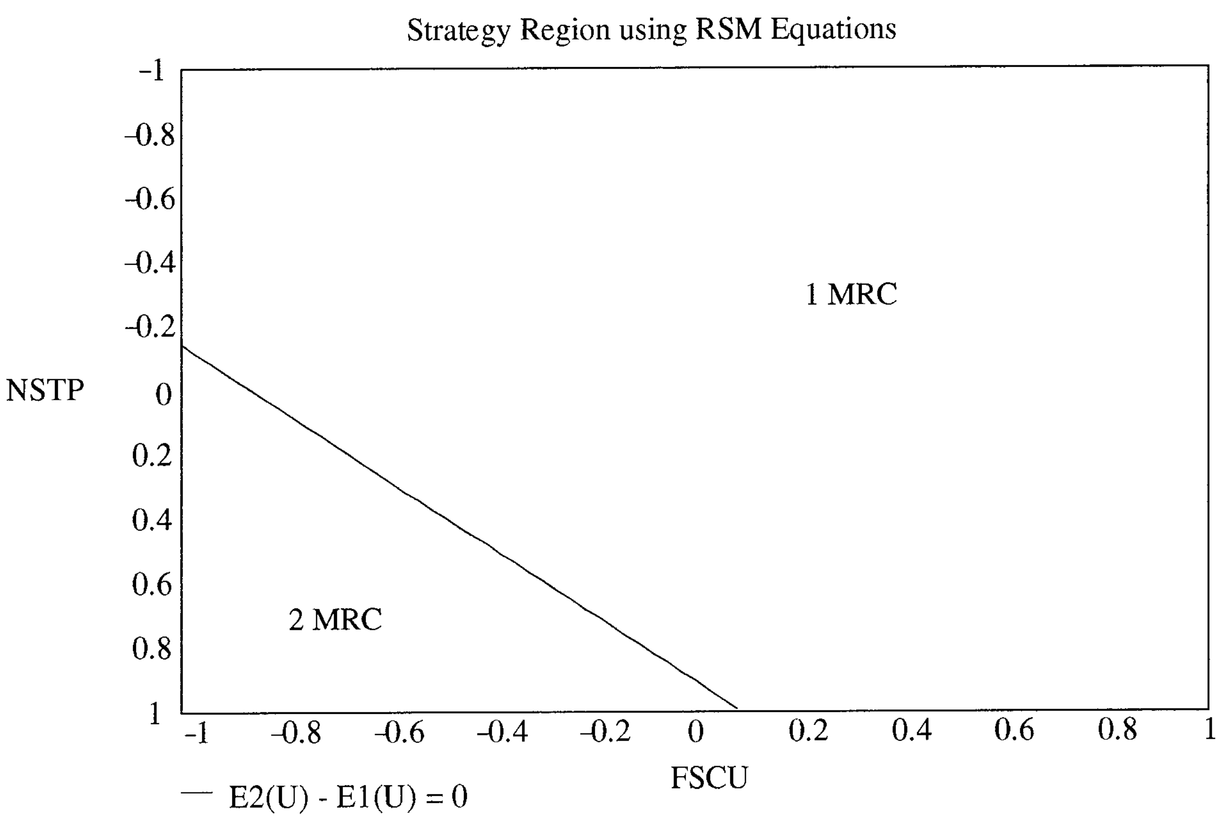


Figure 3.6 Decision Policy Based on Perturbations of FSCU and NSTP (1 or 2 MRC)

Here, the equations are evaluated across the region of interest. The alternatives are compared to generate a strategy region graph like that shown previously in Figure 3.4. The result shows that there are only two optimal options in this region (1 and 2 MRC). Therefore, the expected utility for the one-half MRC force is not a factor in this region. The strategy region depicted in Figure 3.6 is constructed by evaluating $E_{1MRC}(U) - E_{2MRC}(U) = 0$.

IV. Analysis

Initial Solution

The initial solution to the uncertain force structure model corresponds to the maximum expected utility across the force structure alternatives. This maximum expected utility does little to explain the consequences of such a decision. The only clear result is that the optimal decision policy is expected to give the best overall utility as defined in the utility function.

Using the original utility function in Equation (3-3) and solving the influence diagram yields an expected utility of 0.77. This result is based on using an initial one MRC force structure as seen in the decision policy attached in Appendix A. The optimal force structure buildup is dependent on predicted future threats and is shown in Table 4.1.

Table 4.1 shows that a buildup to a two MRC force is needed when the intelligence prediction of the national survival threat is upgraded to high. While, a one MRC force is still optimal if the intelligence prediction of the national survival threat is low. In the case where the national survival threat is predicted to be medium, the predicted threat to national interests should be considered. All of the optimal one MRC regions were equal to the one-half MRC expected utilities for the force structure buildup decision. This is due to the model structure, where reductions in force are not an option. Therefore, the one MRC force would be the solution in these areas.

It can be assumed that the future decision to buildup the force structure will be continually assessed. In this case, the influence diagram would be restructured and reevaluated based on the threat predictions at that time. With this in mind, the sensitivity analysis is focused on the initial force structure decision.

PREDICTED THREAT TO NATIONAL INTERESTS

PREDICTED THREAT TO NATIONAL SURVIVAL		HIGH	MEDIUM	LOW
	HIGH	TWO MRC 0.627	TWO MRC 0.658	TWO MRC 0.664
	MEDIUM	TWO MRC 0.743	ONE MRC* 0.798	ONE MRC* 0.811
	LOW	ONE MRC* 0.754	ONE MRC* 0.816	ONE MRC* 0.828

Table 4.1 Optimal Decision Policy for the Force Structure Buildup Alternatives given a One MRC Initial Force Structure

*** These expected utilities were tied with the one-half MRC option**

Discussion of One-way Sensitivity Analysis

The objective of the one-way sensitivity analysis is to identify significant factors and their impact on the optimal policy. The first technique applied to generate this one-way analysis was the tornado diagram. The tornado diagram was produced in DPL^{TM}, which varied one factor at a time and evaluated the model at the base case and the endpoints. This required 23 additional runs of the model (the endpoints of 11 factors plus the base case). The resulting tornado diagram, shown in Appendix E, indicates that changes in seven of the eleven factors would affect the optimal policy.

Cost weight had the greatest impact on the overall expected value, but did not impact the optimal decision. Therefore, the weights of the three individual utility functions are not critical to the overall decision in this range of interest. The next seven variables prove to be the most interesting. changing the decision policy with perturbations in the range of -0.1 to 0.1. These significant variables are listed here in descending order of impact on the overall utility: FSCU, NSTP, NITP, FSAP, NIP, NIU, and NIIP.

It is interesting to note that NSP, NSU, and NSIP are not significant in the range evaluated. This is counterintuitive when considering that national survival utility is weighted twice as much as the other two individual utilities. Apparently, the national survival utility and national survival probabilities could not be changed enough to make an impact. This is most likely due to the perturbations affecting only the marginal outcome, since secure and insecure are assigned utilities of 1 and 0, respectively. National interests utility and probabilities (NIU, NIP) are perturbed similarly, but the probability of the actual national interests threat (NITP) of being medium or high was considerably more than for the actual national survival threat (NSTP). Table 3.1 shows the actual threat probabilities for national survival and national interests. Since the threat is less likely to become a factor for national survival, the intelligence prediction of national survival threat probabilities (NSIP) is also insignificant in this range. Therefore, these variables would be dropped from further analysis in a standard sensitivity analysis.

It is important to realize that tornado diagrams show changes in decision policy (either decision), not just the initial decision. This fact will become clear when comparing the strategy regions at the end of this chapter with the tornado diagram . The analyst should run the model at specific points of interest to clarify the information gained from the tornado diagram. This will require some additional runs of the model, but should ensure that the correct conclusions are drawn. Also, since the tornado diagram doesn't specify where the decision policy changes, it is necessary to systematically narrow the range to pinpoint this crossover. Again this requires a number of additional runs of the model, depending on the accuracy required.

In comparison to the tornado diagram, the response surface approximation for the overall utility required only twelve runs. The twelve runs are those generated from the Plackett-Burman design shown in Table 3.11. The equation generated shows that the most

$$\begin{aligned} E(U) = 0.776614 - 0.01629 * NSTP + 0.018088 * FSCU - 0.04031 * CW \\ -0.00691 * NSTP * FSCU + 0.00781 FSCU * CW \end{aligned} \qquad (4\text{-}1)$$

significant factor in predicting the overall expected utility is the Cost Weight (CW). The top three variables in the tornado diagram are ranked in the same order in this equation, providing some validation of the method. Here, "significance" refers to the effect that a particular factor has on predicting the expected utility of the optimal decision policy. However, this equation does nothing to identify that policy. Although it is comforting to know that it is easy to generate a parsimonious model, this equation does little to help in the sensitivity analysis. The fact that this expected utility is the maximum of the three alternative's expected utilities provides access to more in-depth insight. The capability to retrieve the alternative's expected values is made possible by DPL^{TM}. The expected utility of each of the three alternatives is found in the Decision Policy output as seen in Appendix A (1:306-309).

The three alternative's responses were gathered for all of the 12 runs plus the original base case and regressed one at a time against the design matrix. This data, along with the augmented Plackett-Burman design matrix is attached in Appendix B. As discussed in Chapter III, the resulting equations are highly significant in a statistical sense. The equations are listed below along with a ranked list of significant factors to aid the reader in the following analysis. These equations are still in the coded variable form.

$E(U)_{2MRC} = 0.75 - 0.05 * CW$

Significant factors:

- Cost Weight (CW)

$$E(U)_{1MRC} = 0.771733 - 0.02365 * NSTP - 0.0156 * NITP + 0.025 * FSCU$$
$$-0.03726 * CW + 0.0058 * FSCU * CW$$

Significant factors:

- Cost Weight (CW)
- Force Structure Cost Utility (FSCU)
- National Survival Threat Probability (NSTP)
- National Interests Threat Probability (NITP)

$$E(U)_{1/2MRC} = 0.724785 - 0.03329 * NSTP - 0.02035 * FSAP - 0.02033 * CW + 0.006069 * NSTP * CW$$

Significant factors:

- National Survival Threat Probability (NSTP)
- Force Structure Available Probability (FSAP)
- Cost Weight (CW)

It is a simple exercise to plug in the variables at the base case, where all of our coded variables are zero. From this, it is established that the maximum of the three alternatives is the $E(U)_{1MRC}$. The one-way sensitivity analysis can then be accomplished by varying one factor at a time and comparing the three expected utilities. This point of view would indicate that there are only five significant variables: CW, NSTP, FSCU, FSAP, and NITP. Obviously, there is some information lost at this point due to the approximation. The question is how much. Upon closer evaluation of the tornado diagram, there are only two variables that affect the initial force structure decision. They are FSCU and NSTP. The rest of the variables that show a change in decision policy affect only the force structure buildup. With this in mind, the information lost may not be as critical as it initially appears. In this sense, our coefficients of regression have picked out the most significant factors in predicting the optimal initial force structure.

Table 4.2 shows the results of comparing the three alternative's RSM equations when the factors are perturbed one at a time. The calculations are performed in a spreadsheet by evaluating:

$$\text{Maximum}\left(E(U)_{2MRC}, E(U)_{1MRC}, E(U)_{1/2MRC}\right) \tag{4-2}$$

This calculation is accomplished with logical statements producing either 2, 1, or 0.5 as the output at each factor setting. The results are the same as the tornado diagram. The only two factors which change the initial force structure decision are NSTP and FSCU. Upon further study of the tornado diagram, it was found that the decision policy change-over point predicted by the approximations agree to within 0.01 of the factor perturbation. This level of accuracy is far beyond that required for the purpose of sensitivity analysis in this model. The cost, in terms of model runs, is the 12 Plackett-Burman design runs plus the 2^3 and 2^4 full factorial design runs. These 36 runs exceed the 23 runs needed for a basic

Range of perturbation

	CW Others	0.2 -0.1	0.21 -0.08	0.22 -0.06	0.23 -0.04	0.24 -0.02	0.25 0	0.26 0.02	0.27 0.04	0.28 0.06	0.29 0.08	0.3 0.1
	CW	1	1	1	1	1	1	1	1	1	1	1
FACTOR	**NSTP**	1	1	1	1	1	1	1	1	1	1	2
	FSCU	2	1	1	1	1	1	1	1	1	1	1
	NITP	1	1	1	1	1	1	1	1	1	1	1
	FSAP	1	1	1	1	1	1	1	1	1	1	1

Table 4.2 One-way Sensitivity Analysis using RSM equations

tornado diagram, but the tornado diagram analysis required an additional 100 or more runs to clarify the results. RSM's frugal use of model runs has only begun to be identified, since no further runs are required to utilize the RSM analysis. The RSM equations can be used in the current form to predict a response to any number of changes in the parameters across the evaluated design region. The usefulness of the approximations formulated using RSM are seen again in the two-way analysis included in the next section.

The analysis of varying one factor at a time indicates that there are only two factors which effect the optimal initial force structure decision in the region of interest. These factors are the force structure cost utility (FSCU - Table 3.6) and the national survival threat probability (NSTP - Table 3.1). The initial force required is increased (1 to 2 MRC force), when the force structure cost utilities are decreased. Remember, only the utilities between 0 and 1 are perturbed. In essence, the cost of starting with a force less than 2 MRC and then building up has gone up when you decrease FSCU. Since these option's utilities are getting closer to zero (the most expensive case), there is less difference in cost between a 2 MRC force and a lesser force.

On the other hand, positive perturbations in the actual national survival threat probability (NSTP) shift the optimal initial force structure decision from a 1 to a 2 MRC force. This is sensible, since there is a increasing probability of the national survival threat being high with such a perturbation. Individually, the effects of changing these two significant factors make sense, but what about changes in both?

Discussion of Two-way Sensitivity Analysis

Standard sensitivity analysis leads to an examination of two-way perturbations. The obvious factors of interest are those that proved significant in the one-way analysis. As discussed in Chapter III, DPL^{TM} rainbow diagrams are used to collect the data required for the two-way sensitivity analysis. An example of the rainbow diagram is attached in Appendix C. This rainbow diagram required eleven runs of the model. The model was evaluated at eleven intervals of FSCU, ranging from -0.1 to 0.1. NSTP was set at 0.1 for these runs. It would take eleven such rainbow diagrams to complete a strategy region as seen in Table 3.10. The total cost, in terms of model runs, is 121 per strategy region. In addition, the strategy region must be generated by pulling the data off of each of the eleven rainbow diagrams. This could amount to a lot of runs and time if there were a number of significant factors involved. In the case where there are five significant factors, it would take:

$$5!/(2!*(5-2)!) = 10 \text{ strategy regions}$$

These ten strategy regions would demand 1210 model runs plus the time spent pulling 1210 data points off of the rainbow diagrams. Another disadvantage of using the rainbow diagram is that the decision change is shown, but the actual decision is not specifically identified. This requires model runs to evaluate what the decision has changed to. At this point in the analysis, the initial method required nearly 250 model runs as compared to RSM's 36. Obviously, RSM can lead to a significant savings in time and model runs. The next step is to figure out the cost of this reduction, in terms of accuracy.

There were several strategy regions generated with both methods for comparison. The results were similar from a accuracy viewpoint. The RSM method was accurate to within 0.005 of expected value results and agreed with the rainbow diagram based strategy regions plus or minus 0.01 of the decision crossover-point. These strategy regions would be used to define trends and to locate areas were the decision policy is sensitive to changes in two variables. Therefore, this accuracy is sufficient in such sensitivity analysis.

Overall, the standard sensitivity analysis is more accurate, but requires an order of magnitude more runs of the model. The efficiency of the RSM technique would be more and more pronounced as the model increases in complexity.

V. Conclusion and Recommendations

The creation of low cost microcomputer software packages, such as DPL™, has opened the doors of decision analysis to new users. This software provides the analyst and decision-maker an easily accessible and flexible tool to work with. DPL™ furnishes a user-friendly interface, aesthetically pleasing output and powerful computational features. Foremost is the ability to formulate, display and solve Influence Diagrams. Influence Diagrams are an excellent tool for model formulation, but just as importantly, they are robust communication tools. They are unmatched in the ability to convey the analyst's understanding of the problem at hand without the confusing detail of other methods. And after the correct problem has been modeled, the solution is close at hand.

Once the initial solution is found, sensitivity analysis becomes the focus of the analysis. It is here, that the standard sensitivity analysis options become restrictive. DPL's™ primary sensitivity tool is the tornado diagram. The tornado diagram is an excellent communication tool to present significant factors and their effects, when perturbed one at a time. A problem with the tornado diagram's created in DPL™, is the limited capability to identify specific data on the factors effects. However, this data is available with additional runs of the model. This requirement for extra clarifying information became a trend and will certainly impact the analysis of any substantial decision model. Beyond the tornado diagram, the software does little to extend sensitivity analysis.

At this point, many problems require some two-way sensitivity analysis. There are three options that might be considered standard for two-way analysis in decision problems. The first would be to run the model at all of the possible combinations of the significant factors. The DPL™ application of this would include building an array of the possible settings and programming the software to run the model at each setting. Obviously, this becomes a considerable task for many models. Next, a good method for simple problems, is to create the equation produced by folding back a decision tree and evaluating it at the settings of interest. This method requires no further runs of the model and produces perfect results. Here, the problem lies in a almost infeasible amount of time to produce such an equation. As mentioned in Chapter III, the three equations in the force structure problem would require almost 50,000 arithmetic operators each. Even if one could type 50 words (variables plus the operators) per minute, it would take over 16 hours to type in each equation into a spreadsheet. There must be a better way and there is. The technique chosen for this analysis is not standard, but took advantage of the options available in the software. This technique used the rainbow diagrams as discussed in Chapter III to run the model over the desired range of variables to compare two factor effects. Again, this could be a substantial undertaking for complex models with many significant factors or when the analysis requires a greater level of accuracy.

Overall, the options discussed above require many additional model runs and/or significantly large, sometimes infeasible, time demands. The recommendation here is to use the software for the analysis up to the tornado diagram. After which, response surface methodology is highly effective.

The estimating equations produced by RSM were highly explanatory, which is stunning considering only 36 model runs are needed to produce them. Using only one strategy region, this yields a overall reduction in computer runs of 93%. The reduction is potentially much higher, considering the bulk of the additional runs were needed for clarification of results and the development of only one strategy region. The cost of this reduction is in an acceptable loss of accuracy. The incredible savings in runs leaves ample room to make a small number of runs for verification of the results. In the end, the RSM technique proved to be a highly successful venture.

Overall, RSM provided an effective and efficient sensitivity tool for this decision analysis problem. It limited the model runs to a minimum and offers highly accurate results over the design region. In addition, the use of the RSM equations facilitates the construction of numerous strategy regions with no additional model runs required. Finally, the regression coefficients yield insights previously unavailable from standard sensitivity analysis.

This study has shown that response surface methodology can be used effectively in the sensitivity analysis of decision analysis problems. More importantly, RSM significantly reduces the time and resources required to produce sensitivity information. This savings in time is well worth the loss of accuracy and affords a great deal more insight into the problem at hand. This study suggests the potential for wide use of RSM in the field of Decision Analysis.

Suggestions for Further Study

As with any thesis, time constraints limit the effort much sooner than potential research areas. There are three areas that show the potential for further study, but could not be accomplished in the time allowed. The first involves the measurement of uncertainty in terms of the optimal decision policy. There needs to be a quantifiable measure of risk, involving the potential for failure, even though the optimal decision was made. It would seem that the probability of the future threat exceeding the force structure recommended can be calculated. This would involve backing out the probability of actual threats and the intelligence prediction given these threats. It would also have to take into account the buildup force available to meet an upgraded threat, if required. The resulting measurement of risk would aid the decision-maker in drawing conclusions and making future evaluations of the value of information.

Next, the sensitivity of the second decision on the force structure buildup could be analyzed in a similar fashion. This would include RSM equations on these nine possibilities and how they react to perturbations in the input variables. This should give the analyst the tools to evolve strategy regions that are broken down into subdivisions of the initial decision. Again, the decision-maker would benefit from the additional information both now and for future evaluations of the model.

Last of all, three-way sensitivity information would be easy to calculate from the RSM equations, but difficult to present and interpret. There may be a need to come up with a three dimensional form of the strategy region including three input variables.

APPENDIX A: Decision Policy for the Initial Solution

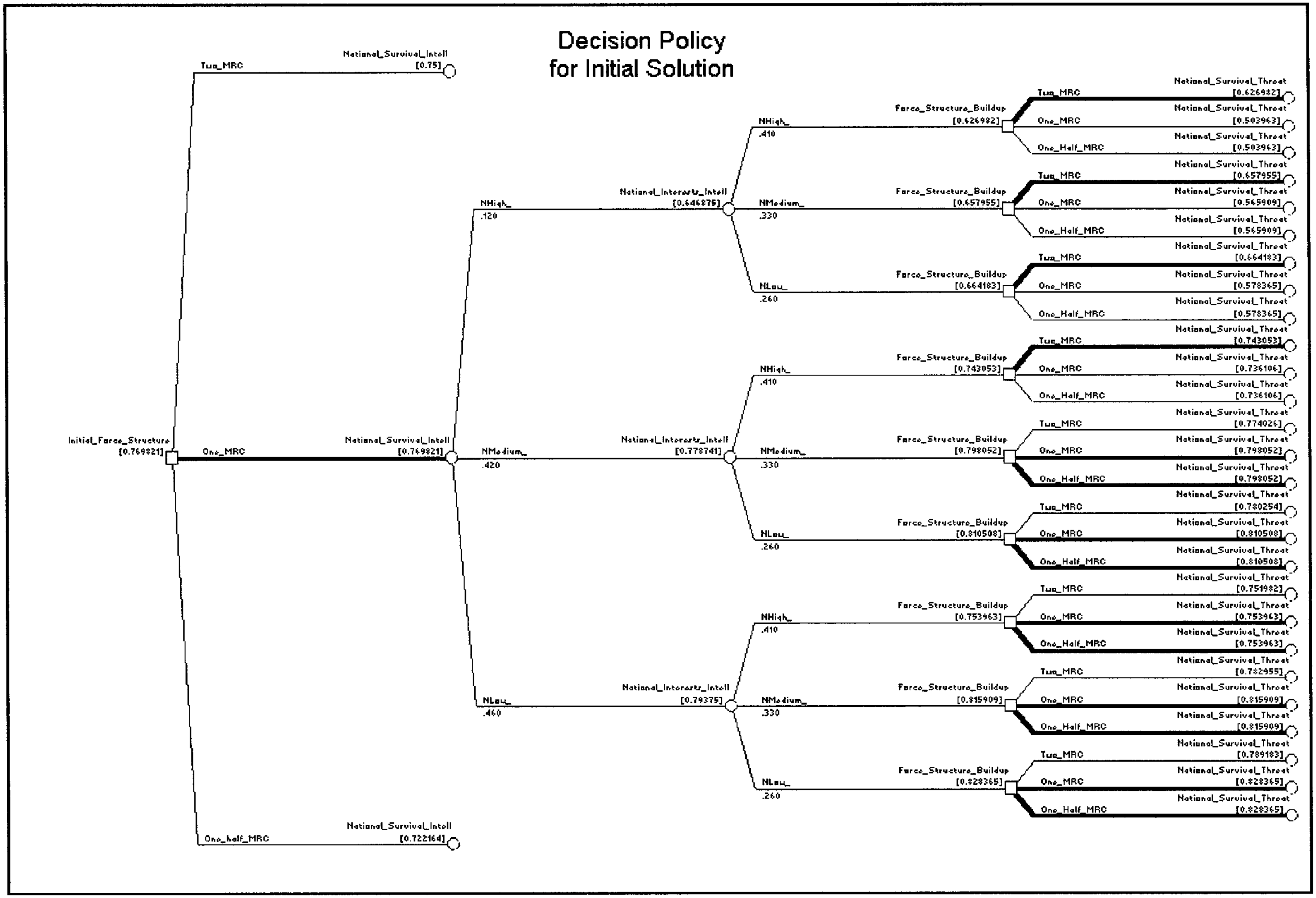


Figure A.1 Decision Policy for Initial Solution

APPENDIX B: Plackett-Burman Design used in RSM Analysis

														INIT FORCE			
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**max[E(U)]**		**2 MRC**	**1 MRC**	**1/2 MRC**
RUN	**I**	**NSIP**	**NIIP**	**NSTP**	**NITP**	**NSP**	**NSU**	**NIP**	**NIU**	**FSAP**	**FSCU**	**CW**	**Y**		**Y1**	**Y2**	**Y3**
1	1	1	1	-1	1	1	1	-1	-1	-1	1	-1	0.835704	**1**	0.8	0.835704	0.81232
2	1	-1	1	1	-1	1	1	1	-1	-1	-1	1	0.700124	**1**	0.7	0.700124	0.695428
3	1	1	-1	1	1	-1	1	1	1	-1	-1	-1	0.8	**2**	0.8	0.746479	0.710742
4	1	-1	1	-1	1	1	-1	1	1	1	-1	-1	0.814776	**1**	0.8	0.814776	0.74712
5	1	-1	-1	1	-1	1	1	-1	1	1	1	-1	0.825202	**1**	0.8	0.825202	0.72728
6	1	-1	-1	-1	1	-1	1	1	-1	1	1	1	0.76432	**1**	0.7	0.764932	0.71068
7	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	0.81264	**0.5**	0.7	0.8116	0.81264
8	1	1	1	-1	-1	-1	1	-1	1	1	-1	1	0.745588	**1**	0.7	0.745588	0.720013
9	1	1	1	1	-1	-1	-1	1	-1	1	1	-1	0.821544	**1**	0.8	0.821544	0.70688
10	1	-1	1	1	1	-1	-1	-1	1	-1	1	1	0.711047	**1**	0.7	0.711047	0.695782
11	1	1	-1	1	1	1	-1	-1	-1	1	-1	1	0.7	**2**	0.7	0.671044	0.61597
12	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0.8264	**1**	0.8	0.8264	0.78976
13	1	0	0	0	0	0	0	0	0	0	0	0	0.76982	**1**	0.75	0.769821	0.722164

Table B.1 The Three Force Options Expected Utility Response vs Plackett-Burman Design

APPENDIX C: Rainbow Diagram used to Create Strategy Region

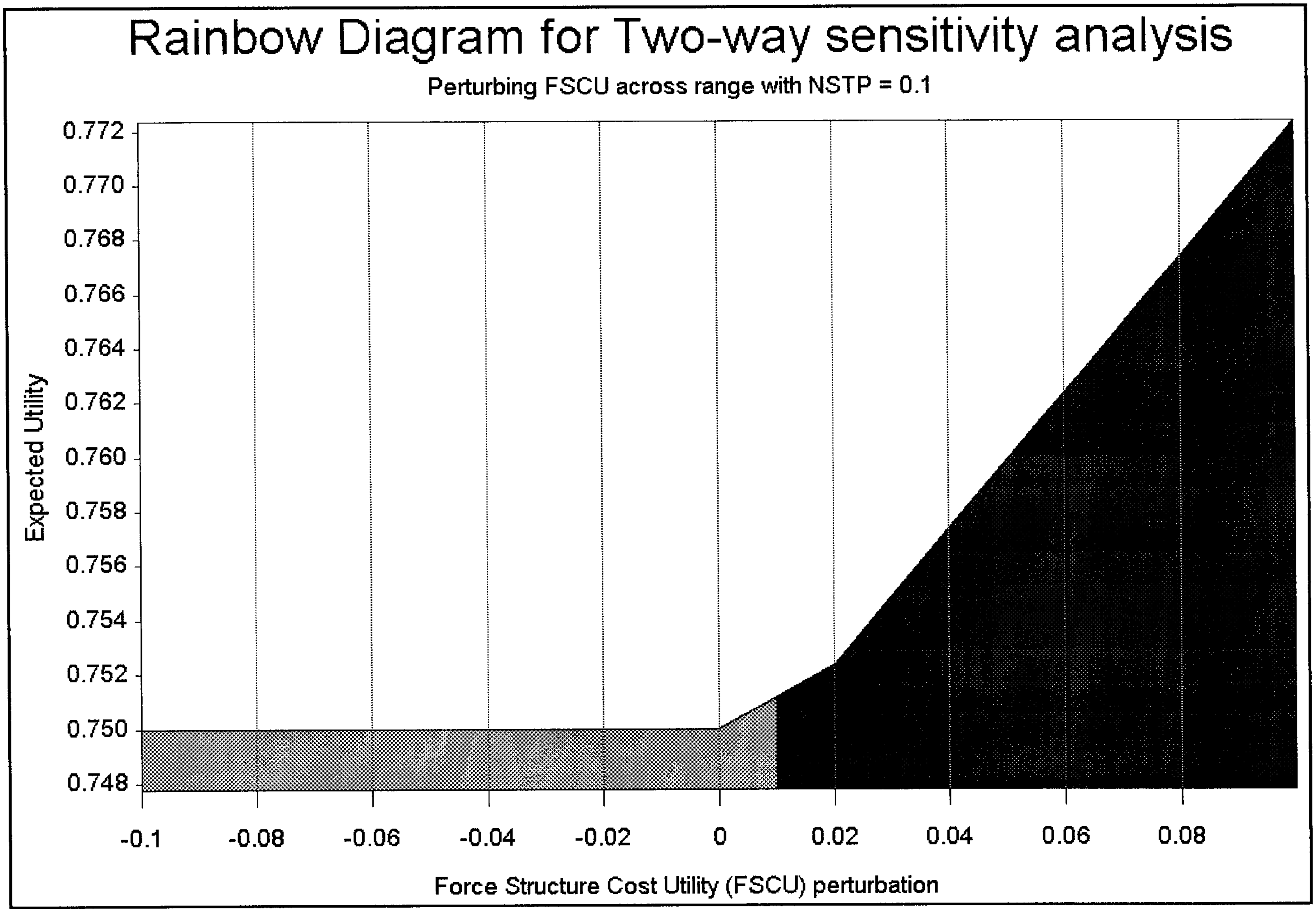


Figure C.1 Example of Rainbow diagram Used to Create Strategy Regions

APPENDIX D: Strategy Region Comparison for Model vs RSM

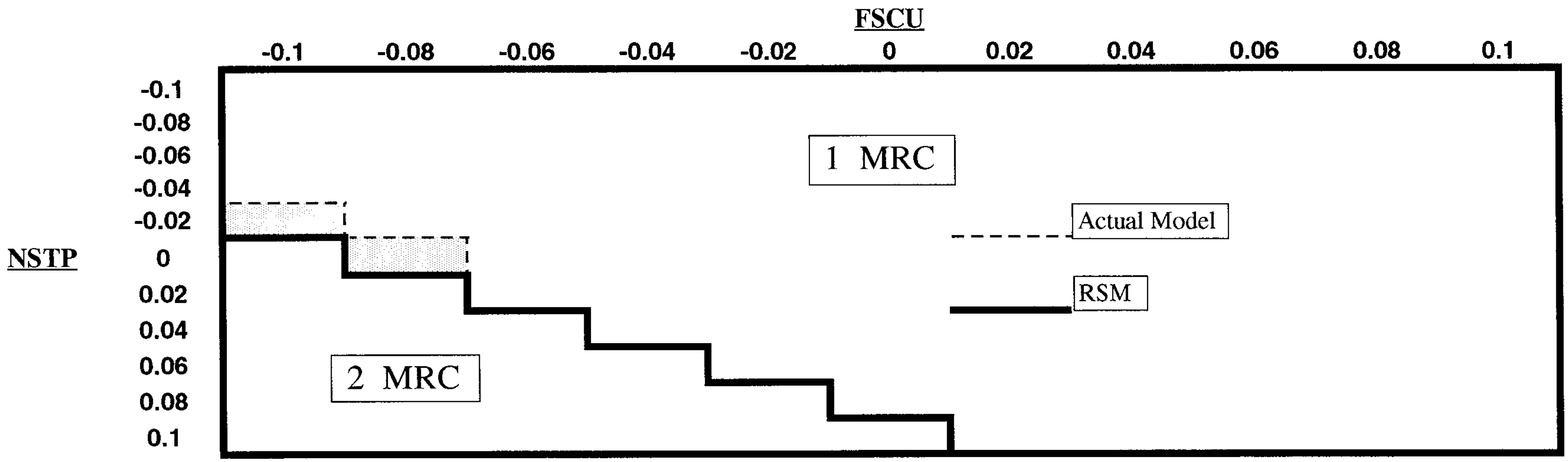
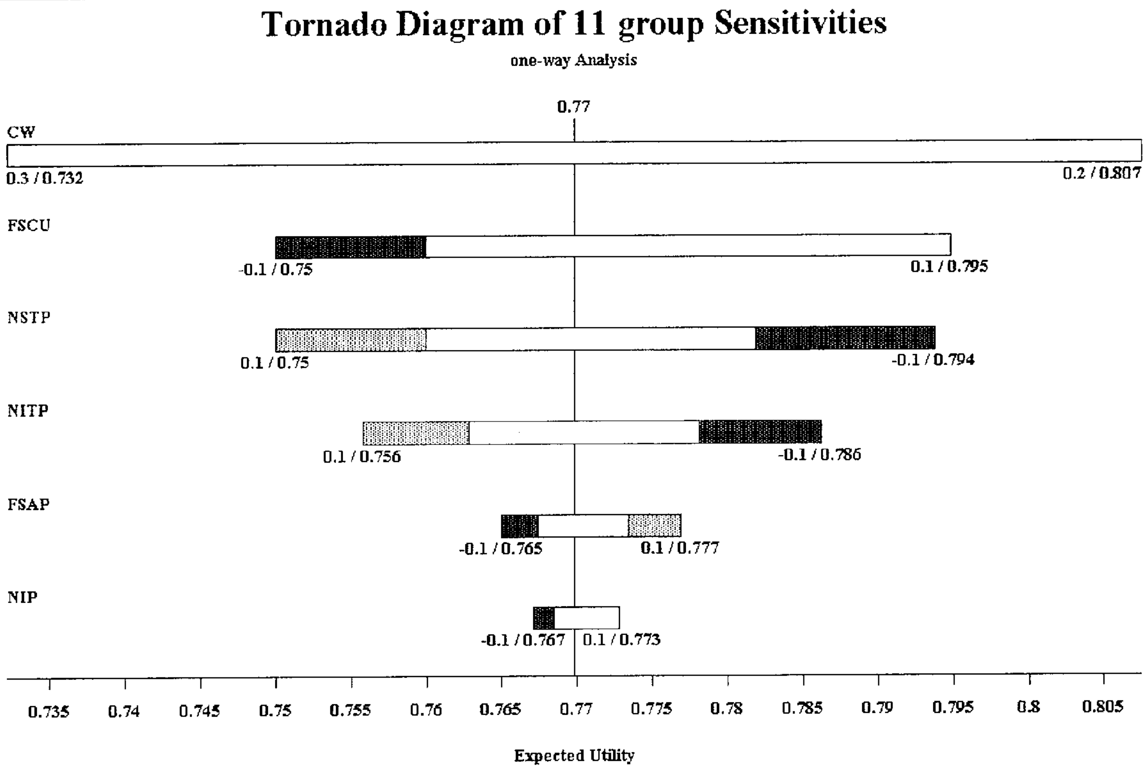


Table D.1 Comparison of Strategy Regions for Optimal Initial Force Structure Decision based on Perturbations in FSCU and NSTP

*Shaded area indicates error of RSM approximation

APPENDIX E: Tornado Diagram of 11 Factor Groups



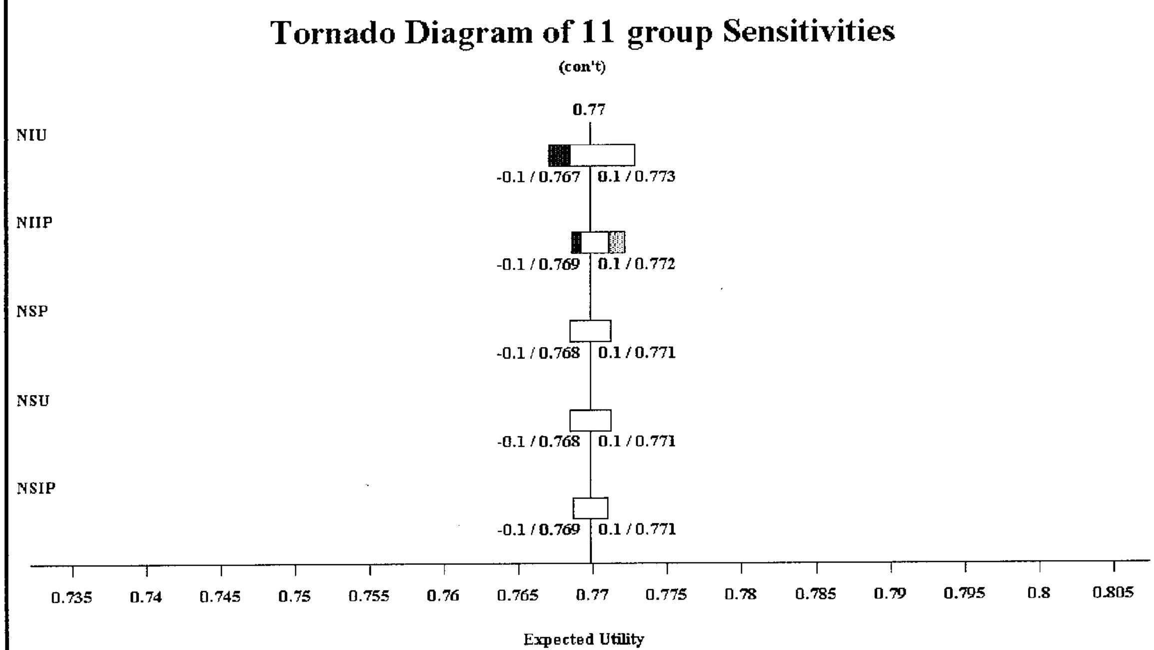


Figure E.1 Tornado Diagram of 11 Groups Perturbed One at a Time

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13. ABSTRACT *(Maximum 200 words)*
The purpose of this study is to evaluate response surface methodology as a sensitivity analysis tool in the area of decision analysis. The advent of low-cost personal computer software, such as DPL™, has created an accessible tool with the ability to frame and solve influence diagrams for decision problems. This study provides a comparison of current sensitivity analysis techniques vs those made possible through response surface methodology (RSM). Sensitivity analysis alternatives are demonstrated on a decision problem concerning the evaluation of force structure options for the Department of Defense. Sensitivity analysis is performed on both one-way and two-way perturbations of input variables. The most significant contribution of this thesis is the effectiveness of RSM techniques for the sensitivity analysis of a decision problem. RSM is proven to be an effective and highly efficient approach. The combination of two analysis tools, the influence diagram solver and the RSM sensitivity analysis, has effectively saved the decision-maker valuable resources and increased the information made available. This capability will enhance the analysis of uncertainty in decision problems.

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